Revisiting the quasinormal modes of the Schwarzschild black hole: Numerical analysis

Luis Alex Huahuachampi Mamani (UFRB)
in collaboration with
Lucas Sanches, Vilson Zanchin (UFABC), and Angel Masa (U. de Cartagena)

Eur.Phys.J.C 82 (2022) 10, 897

VII Join Meeting of Graduate Students Physics - Física/UNSA

the 10th of November, 2022
Arequipa - Perú

Contents

Einstein equations

- Einstein's equations proposed em 1915 are

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

- **Schwarzschild' solution:** spherically solution found by Karl Schwarzschild in 1916. The metric is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2, \quad \text{onde} \quad f(r) = 1 - \frac{2M}{r^2}. \tag{2}$$

- **Perturbation equations:** the radial equation can be written in the Schrödinger-like form

$$\frac{d^2\psi_s(r)}{dr_*^2} + \left(\omega^2 - V_s(r)\right)\psi_s(r) = 0. \tag{3}$$

Perturbation equations - Bosonic fields

The potential of the Schrödinger-like equation for bosonic fields is given by (see for instance [Berti-Cardoso-Starinets, 2009])

$$V_s(r) = f(r)\left(\frac{\ell(\ell+1)}{r^2} + (1-s^2)\frac{2M}{r^3}\right). \tag{4}$$

After applying a few transformations the differential equation suitable to apply the pseudo-spectral and AIM methods is

$$\begin{aligned}\Big(u(\ell(\ell+1)-s^2u)-4\,i\lambda-16\,u(1+u)\lambda^2\Big)\phi_s(u)\\ +\left(u^3+4\,i\,u(1-2u^2)\lambda\right)\phi_s'(u)-(1-u)u^3\phi_s''(u)=0,\end{aligned} \tag{5}$$

where $r = 1/u$ and $\lambda = \omega M$. Note that $u \in [0, 1]$.

Perturbation equation - spin 1/2 field

For spin 1/2 field the potential of the Schrödinger-like equation is given by (see [Cho, Phys. Rev. D 68, 024003] and Shu and Shen, Phys. Lett. B 619, 340])

$$V_{1/2} = \frac{(1+\ell)\sqrt{r-2M}}{r^{7/2}}\left((1+\ell)\sqrt{r(r-2M)} + 3M - r\right) \tag{6}$$

After applying a few transformation the differential equation suitable to apply the pseudo-spectral and AIM methods is

$$\Big(u^3 + u(1+\ell)(1+\ell-\sqrt{1-u}) + \frac{u^2}{2}((1+\ell)(3\sqrt{1-u}-4) - 2\ell^2) - 4\,i\,(1-u)\lambda$$
$$- 16u(1-u^2)\lambda^2\Big)\phi_{1/2}(u) + \left(u^3(1-u) + 4\,i\,u(1-u-2u^2+2u^3)\lambda\right)\phi'_{1/2}(u)$$
$$- u^3(1-u)^2\phi''_{1/2}(u) = 0.$$

where $r = 1/u$ and $\lambda = \omega M$. Note that $u \in [0,1]$. For spin 3/2 and 5/2 fields we did the same analysis, we did not present here because their expressions are huge.

The pseudo-spectral method - short review

The idea behind the pseudo-spectral method is to rewrite the regular function $\phi(u)$ in a base composed by cardinal functions $C_j(u)$ and the Gauss-Lobato grid, in the form (for details see [Jansen, Eur. Phys. J. Plus 132, 546])

$$\phi_s(u) = \sum_{j=0}^{N} g(u_j)\, C_j(u); \qquad u_i = \frac{1}{2}\left(1 \pm \cos\left[\frac{i}{N}\pi\right]\right), \quad i = 0, 1, 2, \cdots, N \tag{7}$$

Then, the matrix representation of the quadratic eigenvalue problem can be written as

$$\left(\tilde{M}_0 + \tilde{M}_1\lambda + \tilde{M}_2\lambda^2\right) g = 0; \quad \rightarrow \quad (M_0 + M_1\,\lambda)\cdot \vec{g} = \mathbb{0}. \tag{8}$$

$$C_j(u) = T_j(u), \qquad C_j(u) = \frac{2}{Np_j}\sum_{m=0}^{N}\frac{1}{p_m}T_m(u_j)\,T_m(u), \quad \begin{cases} p_0 = 2, \\ p_N = 2, \\ p_j = 1. \end{cases} \tag{9}$$

The asymptotic iteration method - short review

The second order differential equation can be written in the form [H. Ciftci et al,. 2003, Journal of Physics A: Mathematical and General]

$$y''(x) - \lambda_0(x)y'(x) - s_0(x)y(x) = 0 \tag{10}$$

where $\lambda_0(x) \neq 0$ and $s_0(x)$ are C_∞. A general solution of Eq. (10) can be written in the form

$$y(x) = \exp\left(-\int \alpha \mathrm{d}t\right) \times \left[C_2 + C_1 \int^x \exp\left(\int^t (\lambda_0(\tau) + 2\alpha(\tau))\mathrm{d}\tau\right) \mathrm{d}t\right] \tag{11}$$

if for some $n > 0$ the condition

$$\delta \equiv s_n\lambda_{n-1} - \lambda_n s_{n-1} = 0 \tag{12}$$

is satisfied. However, for computing QNMs it is necessary a modification of this procedure in order to circumvent numerical problems.

The asymptotic iteration method - short review

Instead, the coefficients are expanded around an arbitrary point, say $x = \xi$, (see [Cho, Cornell, Doukas, Huang, and Naylor, Adv. Math. Phys.2012, 281705])

$$\lambda_n(\xi) = \sum_{i=0}^{\infty} c_n^i (x-\xi)^i; \qquad s_n(\xi) = \sum_{i=0}^{\infty} d_n^i (x-\xi)^i. \tag{13}$$

The quantization condition becomes

$$\delta \equiv d_n^0 c_{n-1}^0 - d_{n-1}^0 c_n^0 = 0. \tag{14}$$

where

$$c_n^i = (i+1)c_{n-1}^{i+1} + d_{n-1}^i + \sum_{k=0}^{i} c_0^k c_{n-1}^{i-k}; \qquad d_n^i = (i+1)d_{n-1}^{i+1} + \sum_{k=0}^{i} d_0^k c_{n-1}^{i-k}. \tag{15}$$

A package implemented in *Julia* can be found in:
https://github.com/lucass-carneiro/QuasinormalModes.jl.

Spin 0

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations	Ref. [16]	Ref. [17]
0	0	$\pm 0.110455 - 0.104896i$	$\pm 0.110455 - 0.104896i$	$\pm 0.110455 - 0.104896i$	$0.1046 - 0.1152i$	$\pm 0.1105 - 0.1008i$
1	0	$\pm 0.292936 - 0.097660i$	$\pm 0.292936 - 0.097660i$	$\pm 0.292936 - 0.097660i$	$0.2911 - 0.0980i$	$\pm 0.2929 - 0.0978i$
	1	$\pm 0.264449 - 0.306257i$	$\pm 0.264449 - 0.306257i$	—	—	$\pm 0.2645 - 0.3065i$
2	0	$\pm 0.483644 - 0.096759i$	$\pm 0.483644 - 0.096759i$	$\pm 0.483644 - 0.096759i$	$0.4832 - 0.0968i$	$\pm 0.4836 - 0.0968i$
	1	$\pm 0.463851 - 0.295604i$	$\pm 0.463851 - 0.295604i$	$\pm 0.463851 - 0.295604i$	$0.4632 - 0.2958i$	$\pm 0.4638 - 0.2956i$
	2	$\pm 0.430544 - 0.508558i$	$\pm 0.430544 - 0.508558i$	—	—	$\pm 0.4304 - 0.5087i$
3	0	$\pm 0.675366 - 0.096500i$	$\pm 0.675366 - 0.096500i$	$\pm 0.675366 - 0.096500i$	$0.6752 - 0.0965i$	—
	1	$\pm 0.660671 - 0.292285i$	$\pm 0.660671 - 0.292285i$	$\pm 0.660671 - 0.292285i$	$0.6604 - 0.2923i$	—
	2	$\pm 0.633626 - 0.496008i$	$\pm 0.633626 - 0.496008i$	$\pm 0.633626 - 0.496008i$	$0.6348 - 0.4941i$	—
	3	$\pm 0.598773 - 0.711221i$	$\pm 0.598769 - 0.711220i$	—	—	—
4	0	$\pm 0.867416 - 0.096392i$	$\pm 0.867416 - 0.096392i$	$\pm 0.867416 - 0.096392i$	$0.8673 - 0.0964i$	—
	1	$\pm 0.855808 - 0.290876i$	$\pm 0.855808 - 0.290876i$	$\pm 0.855808 - 0.290876i$	$0.8557 - 0.2909i$	—
	2	$\pm 0.833692 - 0.490325i$	$\pm 0.833692 - 0.490325i$	$\pm 0.833692 - 0.490325i$	$0.8345 - 0.4895i$	—
	3	$\pm 0.803288 - 0.697482i$	$\pm 0.803288 - 0.697482i$	$\pm 0.803288 - 0.697482i$	$0.8064 - 0.6926i$	—
	4	$\pm 0.767733 - 0.914019i$	$\pm 0.767679 - 0.914105i$	—	—	—

[16] Shu and Shen, Phys. Lett. B 619, 340 (2005), [17] Konoplya, J. Phys. Stud. 8, 93 (2004).

Spin 1

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations	Ref. [16]	Ref. [17]
0	0	—	—	—	—	—
1	0	$\pm 0.248263 - 0.092488i$	$\pm 0.248263 - 0.092488i$	$\pm 0.248263 - 0.092488i$	$0.2459 - 0.0931i$	$\pm 0.2482 - 0.0926i$
	1	$\pm 0.214515 - 0.293668i$	$\pm 0.214515 - 0.293667i$	—	—	$\pm 0.2143 - 0.2941i$
2	0	$\pm 0.457596 - 0.095004i$	$\pm 0.457595 - 0.095004i$	$\pm 0.457596 - 0.095004i$	$0.4571 - 0.0951i$	$\pm 0.4576 - 0.0950i$
	1	$\pm 0.436542 - 0.290710i$	$\pm 0.436542 - 0.290710i$	$\pm 0.436542 - 0.290710i$	$0.4358 - 0.2910i$	$\pm 0.4365 - 0.2907i$
	2	$\pm 0.401187 - 0.501587i$	$\pm 0.401187 - 0.501587i$	—	—	$\pm 0.4009 - 0.5017i$
3	0	$\pm 0.656899 - 0.095616i$	$\pm 0.656899 - 0.095616i$	$\pm 0.656899 - 0.095616i$	$0.6567 - 0.0956i$	$\pm 0.6569 - 0.0956i$
	1	$\pm 0.641737 - 0.289728i$	$\pm 0.641737 - 0.289728i$	$\pm 0.641737 - 0.289728i$	$0.6415 - 0.2898i$	$\pm 0.6417 - 0.2897i$
	2	$\pm 0.613832 - 0.492066i$	$\pm 0.613832 - 0.492066i$	$\pm 0.613832 - 0.492066i$	$0.6151 - 0.4901i$	$\pm 0.6138 - 0.4921i$
	3	$\pm 0.577919 - 0.706331i$	$\pm 0.577915 - 0.706328i$	—	—	$\pm 0.5775 - 0.7065i$
4	0	$\pm 0.853095 - 0.095860i$	$\pm 0.853095 - 0.095860i$	$\pm 0.853095 - 0.095810i$	$0.8530 - 0.0959i$	—
	1	$\pm 0.841267 - 0.289315i$	$\pm 0.841267 - 0.289315i$	$\pm 0.841267 - 0.289315i$	$0.8411 - 0.2893i$	—
	2	$\pm 0.818728 - 0.487838i$	$\pm 0.818728 - 0.487838i$	$\pm 0.818728 - 0.487838i$	$0.8196 - 0.4870i$	—
	3	$\pm 0.787748 - 0.694242i$	$\pm 0.787748 - 0.694243i$	$\pm 0.787748 - 0.694242i$	$0.7909 - 0.6892i$	—
	4	$\pm 0.751549 - 0.910242i$	$\pm 0.751481 - 0.910301i$	—	—	—

[16] Shu and Shen, Phys. Lett. B 619, 340 (2005), [17] Konoplya, J. Phys. Stud. 8, 93 (2004).

Spin 2

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations	Ref. [16]	Ref. [17]
0	0	—	—	—	—	—
1	0	$\pm0.110455 - 0.104896i$	$\pm0.110455 - 0.104896i$	—	—	—
	1	—	—	—	—	—
2	0	$\pm0.373672 - 0.088962i$	$\pm0.373672 - 0.088962i$	$\pm0.373672 - 0.088962i$	$0.3730 - 0.0891i$	$\pm0.3736 - 0.0890i$
	1	$\pm0.346711 - 0.273915i$	$\pm0.346711 - 0.273915i$	$\pm0.346711 - 0.273915i$	$0.3452 - 0.2746i$	$\pm0.3463 - 0.2735i$
	2	$\pm0.301053 - 0.478277i$	$\pm0.301053 - 0.478277i$	—	—	$\pm0.2985 - 0.4776i$
3	0	$\pm0.599443 - 0.092703i$	$\pm0.599443 - 0.092703i$	$\pm0.599443 - 0.092703i$	$0.5993 - 0.0927i$	$\pm0.5994 - 0.0927i$
	1	$\pm0.582644 - 0.281298i$	$\pm0.582644 - 0.281298i$	$\pm0.582644 - 0.281298i$	$0.5824 - 0.2814i$	$\pm0.5826 - 0.2813i$
	2	$\pm0.551685 - 0.479093i$	$\pm0.551685 - 0.479093i$	$\pm0.551685 - 0.479027i$	$0.5532 - 0.4767i$	$\pm0.5516 - 0.4790i$
	3	$\pm0.511962 - 0.690337i$	$\pm0.511966 - 0.690333i$	—	—	$\pm0.5111 - 0.6905i$
4	0	$\pm0.809178 - 0.094164i$	$\pm0.809178 - 0.094164i$	$\pm0.809178 - 0.094164i$	$0.8091 - 0.0942i$	$\pm0.8092 - 0.0942i$
	1	$\pm0.796632 - 0.284334i$	$\pm0.796632 - 0.284334i$	$\pm0.796632 - 0.284334i$	$0.7965 - 0.2844i$	$\pm0.7966 - 0.2843i$
	2	$\pm0.772710 - 0.479908i$	$\pm0.772710 - 0.479908i$	$\pm0.772710 - 0.479908i$	$0.7736 - 0.4790i$	$\pm0.7727 - 0.4799i$
	3	$\pm0.739837 - 0.683924i$	$\pm0.739836 - 0.683925i$	$\pm0.739837 - 0.683924i$	$0.7433 - 0.6783i$	$\pm0.7397 - 0.6839i$
	4	$\pm0.701516 - 0.898239i$	$\pm0.701398 - 0.898196i$	—	—	$\pm0.7006 - 0.8985i$

[16] Shu and Shen, Phys. Lett. B 619, 340 (2005), [17] Konoplya, J. Phys. Stud. 8, 93 (2004).

Spin 1/2

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations	Ref. [16]	Ref. [18]
0	0	$\pm 0.182963 - 0.096982i$	$\pm 0.182963 - 0.096982i$	$\pm 0.182963 - 0.096824i$	—	—
1	0	$\pm 0.380037 - 0.096405i$	$\pm 0.380037 - 0.096405i$	$\pm 0.380037 - 0.096405i$	$0.3786 - 0.0965i$	$0.379 - 0.097i$
	1	$\pm 0.355833 - 0.297497i$	$\pm 0.355833 - 0.297497i$	$\pm 0.355833 - 0.297497i$	—	—
2	0	$\pm 0.574094 - 0.096305i$	$\pm 0.574094 - 0.096305i$	$\pm 0.574094 - 0.096305i$	$0.5737 - 0.0963i$	$0.574 - 0.096i$
	1	$\pm 0.557015 - 0.292715i$	$\pm 0.557015 - 0.292715i$	$\pm 0.557015 - 0.292715i$	$0.5562 - 0.2930i$	$0.556 - 0.293i$
	2	$\pm 0.526607 - 0.499695i$	$\pm 0.526607 - 0.499695i$	$\pm 0.526607 - 0.499695i$	—	—
3	0	$\pm 0.767355 - 0.096270i$	$\pm 0.767355 - 0.096270i$	$\pm 0.767355 - 0.096270i$	$0.7672 - 0.0963i$	$0.767 - 0.096i$
	1	$\pm 0.754300 - 0.290968i$	$\pm 0.754300 - 0.290968i$	$\pm 0.754300 - 0.290968i$	$0.7540 - 0.2910i$	$0.754 - 0.291i$
	2	$\pm 0.729770 - 0.491910i$	$\pm 0.729770 - 0.491910i$	$\pm 0.729770 - 0.491910i$	$0.7304 - 0.4909i$	$0.730 - 0.491i$
	3	$\pm 0.696913 - 0.702293i$	$\pm 0.696913 - 0.702293i$	$\pm 0.696913 - 0.702293i$	—	—
4	0	$\pm 0.960293 - 0.096254i$	$\pm 0.960292 - 0.096254i$	$\pm 0.960293 - 0.096254i$	$0.9602 - 0.0963i$	$0.960 - 0.096i$
	1	$\pm 0.949759 - 0.290148i$	$\pm 0.949759 - 0.290148i$	$\pm 0.949759 - 0.290148i$	$0.9496 - 0.2902i$	$0.950 - 0.290i$
	2	$\pm 0.929494 - 0.488116i$	$\pm 0.929494 - 0.488116i$	$\pm 0.929494 - 0.488116i$	$0.9300 - 0.4876i$	$0.930 - 0.488i$
	3	$\pm 0.901129 - 0.692520i$	$\pm 0.901129 - 0.692520i$	$\pm 0.901129 - 0.692520i$	$0.9036 - 0.6892i$	$0.904 - 0.689i$
	4	$\pm 0.867043 - 0.905047i$	$\pm 0.867008 - 0.905066i$	$\pm 0.867043 - 0.905047i$	—	—

[16] Shu and Shen, Phys. Lett. B 619, 340 (2005), [18] Cho, Phys. Rev. D 68, 024003 (2003).

Spin 3/2

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations	Ref. [16]	Ref. [23]
0	0	$\pm 0.311292 - 0.090087i$	$\pm 0.311292 - 0.090087i$	$\pm 0.311292 - 0.090087i$	—	$0.3112 - 0.0902i$
1	0	$\pm 0.530048 - 0.093751i$	$\pm 0.530048 - 0.093751i$	$\pm 0.530048 - 0.093751i$	—	$0.5300 - 0.0937i$
	1	$\pm 0.511392 - 0.285423i$	$\pm 0.511392 - 0.285423i$	$\pm 0.511392 - 0.285423i$	—	$0.5113 - 0.2854i$
2	0	$\pm 0.734750 - 0.094878i$	$\pm 0.734750 - 0.094878i$	$\pm 0.734750 - 0.094878i$	$0.7346 - 0.0949i$	$0.7347 - 0.0948i$
	1	$\pm 0.721047 - 0.286906i$	$\pm 0.721047 - 0.286906i$	$\pm 0.721047 - 0.286906i$	$0.7206 - 0.2870i$	$0.7210 - 0.2869i$
	2	$\pm 0.695287 - 0.485524i$	$\pm 0.695287 - 0.485524i$	$\pm 0.695287 - 0.485524i$	—	$0.6952 - 0.4855i$
3	0	$\pm 0.934364 - 0.095376i$	$\pm 0.934364 - 0.095376i$	$\pm 0.934364 - 0.095376i$	$0.9343 - 0.0954i$	$0.9343 - 0.0953i$
	1	$\pm 0.923502 - 0.287560i$	$\pm 0.923502 - 0.287560i$	$\pm 0.923502 - 0.287560i$	$0.9233 - 0.2876i$	$0.9235 - 0.2875i$
	2	$\pm 0.902599 - 0.483957i$	$\pm 0.902599 - 0.483957i$	$\pm 0.902599 - 0.483957i$	$0.9031 - 0.4835i$	$0.9025 - 0.4839i$
	3	$\pm 0.873342 - 0.687024i$	$\pm 0.873343 - 0.687024i$	$\pm 0.873342 - 0.687024i$	—	$0.8732 - 0.6870i$
4	0	$\pm 1.131530 - 0.095640i$	$\pm 1.131530 - 0.095640i$	$\pm 1.131530 - 0.095640i$	$1.1315 - 0.0956i$	$1.1315 - 0.0956i$
	1	$\pm 1.122523 - 0.287908i$	$\pm 1.122523 - 0.287908i$	$\pm 1.122523 - 0.287908i$	$1.1224 - 0.2879i$	$1.1225 - 0.2879i$
	2	$\pm 1.104976 - 0.483096i$	$\pm 1.104976 - 0.483096i$	$\pm 1.104976 - 0.483096i$	$1.1053 - 0.4828i$	$1.1049 - 0.4830i$
	3	$\pm 1.079852 - 0.683000i$	$\pm 1.079852 - 0.683000i$	$\pm 1.079852 - 0.683000i$	$1.0817 - 0.6812i$	$1.0798 - 0.6829i$
	4	$\pm 1.048599 - 0.889113i$	$\pm 1.048596 - 0.889115i$	$\pm 1.048599 - 0.889113i$	—	$1.0484 - 0.8890i$

[16] Shu and Shen, Phys. Lett. B 619, 340 (2005), [23] Chen, Cho, Cornell, and Harmsen, Phys. Rev. D 94, 044052 (2016).

Spin 1/2 and 3/2

It is worth mentioning that we found additional frequencies which are purely imaginary and the same for spin 1/2 and 3/2 fields.

Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations
$-0.250000i$	$-0.250000i$	$-0.250000i$
$-0.500000i$	$-0.500000i$	$-0.500000i$
$-0.750000i$	$-0.750000i$	$-0.750000i$
$-1.000000i$	$-1.000000i$	$-1.000031i$
$-1.2499998i$	$-1.250000i$	$-1.246550i$

Spin 5/2

l	n	Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations
0	0	$\pm 0.462727 - 0.092578i$	$\pm 0.462727 - 0.092578i$	$0.462727 - 0.092577i$
1	0	$\pm 0.687103 - 0.094566i$	$\pm 0.687103 - 0.094566i$	$0.687103 - 0.094566i$
	1	$\pm 0.670542 - 0.285767i$	$\pm 0.670542 - 0.285767i$	$0.670542 - 0.285767i$
2	0	$\pm 0.897345 - 0.095309i$	$\pm 0.897345 - 0.095309i$	$0.897345 - 0.095309i$
	1	$\pm 0.884980 - 0.287266i$	$\pm 0.884980 - 0.287266i$	$0.884980 - 0.287266i$
	2	$\pm 0.861109 - 0.483113i$	$\pm 0.861109 - 0.483113i$	$0.861109 - 0.483113i$
3	0	$\pm 1.101190 - 0.095648i$	$\pm 1.101190 - 0.095648i$	$1.101190 - 0.095648i$
	1	$\pm 1.091300 - 0.287886i$	$\pm 1.091300 - 0.287886i$	$1.091300 - 0.287886i$
	2	$\pm 1.071999 - 0.482895i$	$\pm 1.071999 - 0.482895i$	$1.071999 - 0.482895i$
	3	$\pm 1.044272 - 0.682307i$	$\pm 1.044272 - 0.682307i$	$1.044272 - 0.682307i$
4	0	$\pm 1.301587 - 0.095829i$	$\pm 1.301587 - 0.095829i$	$1.301587 - 0.095829i$
	1	$\pm 1.293328 - 0.288184i$	$\pm 1.293328 - 0.288184i$	$1.293328 - 0.288184i$
	2	$\pm 1.277107 - 0.482604i$	$\pm 1.277107 - 0.482604i$	$1.277107 - 0.482604i$
	3	$\pm 1.253526 - 0.680366i$	$\pm 1.253526 - 0.680366i$	$1.253526 - 0.680366i$
	4	$\pm 1.223513 - 0.882554i$	$\pm 1.223512 - 0.882553i$	$1.223513 - 0.882554i$

Spin 5/2

We also found purely imaginary frequencies for spin 5/2 field.

Pseudo-spectral I (60 Polynomials)	Pseudo-spectral II (40 polynomials)	AIM 100 Iterations
$-0.125000i$	$-0.125000i$	$-0.125000i$
$-0.375602i$	$-0.375602i$	$-0.378659i$
$-0.626877i$	$-0.626877i$	$-0.623931i$
$-0.878946i$	$-0.878948i$	$-0.907374i$

Conclusions and outlook

- We applied successfully the pseudo-spectral and AIM methods to calculate quasi-normal frequencies of the Schwarzschild black hole for spin 0, 1/2, 1, 3/2, 2 and 5/2 fields.
- For bosonic fields (spin 0, 1, and 2) we found results in agreement with results available in the literature. We did not find any additional frequency.
- For fermionic fields (spin 1/2 and 3/2) we found results in agreement with results available in the literature. We also found additional frequencies which are purely imaginary.
- For spin 5/2 field we found results not available in the literature. We also found frequencies which are purely imaginary.
- In the next stage we are going to address the case of charged and rotating black holes.

Thank you!

Contact Information

- e-mail: luis.mamani@ufrb.edu.br
- Profile in INSPIRE-HEP: https://inspirehep.net/authors/1284231

- Physics graduate program at UESC: PROFISICA