Teorías de gauge a la Utiyama

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Ecuación de Euler-Lagrange

$$S = \int d^4x \, \mathcal{L}_M(\phi, \partial_\mu \phi) \qquad \partial_\mu \phi = \frac{\partial \phi}{\partial x^\mu}$$

$$\delta S = \int d^4x \, \delta \mathcal{L}_M(\phi, \partial_\mu \phi) = 0$$

$$\Rightarrow \int d^4x \left[\frac{\partial \mathcal{L}_M}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)} \delta(\partial_\mu \phi) \right] = 0$$

$$\partial_\mu \left[\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)} \delta\phi \right] = \frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)} \partial_\mu(\delta\phi) + \partial_\mu \left(\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)} \right) \delta\phi$$

$$\int d^4x \left[\frac{\partial \mathcal{L}_M}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)}\right)\right] \delta\phi + \int d^4x \partial_\mu \left[\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)} \delta\phi\right] = 0$$

$$\frac{\partial \mathcal{L}_M}{\partial \phi} - \partial_\mu \left(\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi)}\right) = 0$$

$$\frac{\partial \mathcal{L}_M}{\partial \phi^A} - \partial_\mu \left(\frac{\partial \mathcal{L}_M}{\partial(\partial_\mu \phi^A)}\right) = 0$$

Transformación de gauge Global

$$\phi^A \to \phi^A + \delta\phi^A$$

$$\delta\phi^A = \epsilon^a I^A_{(a)B}\phi^B \qquad a = 1,2,\dots,n$$

$$I^A_{(a)B}$$

$$\left[I_{(a)}, I_{(b)}\right]^A_B = I^A_{(a)C} I^C_{(b)B} - I^A_{(b)C} I^C_{(a)B} = f^c_{ab} I^A_{(c)B}$$

$$f^m_{ab} f^l_{mc} + f^m_{bc} f^l_{ma} + f^m_{ca} f^l_{mb} = 0$$

$$\delta\phi^A = \epsilon^a I^A_{(a)B}\phi^B$$

$$\delta\mathcal{L}_M = \frac{\partial\mathcal{L}_M}{\partial\phi^A}\delta\phi^A + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta(\partial_\mu\phi^A) = 0$$

$$\frac{\partial\mathcal{L}_M}{\partial\phi^A}\delta\phi^A + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\partial_\mu(\delta\phi^A) = 0$$

$$\delta\phi^A = \epsilon^a I^A_{(a)B}\phi^B$$

$$\frac{\partial\mathcal{L}_M}{\partial\phi^A} I^A_{(a)B}\phi^B + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)} I^A_{(a)B}\partial_\mu\phi^B = 0$$

Teorema de Noether

$$\delta\mathcal{L}_M = \frac{\partial\mathcal{L}_M}{\partial\phi^A}\delta\phi^A + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta(\partial_\mu\phi^A) = 0$$

$$\frac{\partial\mathcal{L}_M}{\partial\phi^A}\delta\phi^A - \partial_\mu\left(\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\right)\delta\phi^A + \partial_\mu\left[\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta\phi^A\right] = 0$$

$$\left[\frac{\partial\mathcal{L}_M}{\partial\phi^A} - \partial_\mu\left(\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\right)\right]\delta\phi^A + \partial_\mu\left[\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta\phi^A\right] = 0$$

$$J^\mu = \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta\phi^A$$

$$J^\mu_a = \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}I^A_{(a)B}\phi^B$$

$$\partial_\mu J^\mu_a = 0$$

Transformación de gauge Local

$$\delta\phi^A = \epsilon^a(x) I^A_{(a)B}\phi^B \qquad a = 1,2,\dots,n$$

$$\delta\mathcal{L}_M = \frac{\partial\mathcal{L}_M}{\partial\phi^A}\delta\phi^A + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\delta(\partial_\mu\phi^A) = 0$$

$$\delta(\partial_\mu\phi^A) = \partial_\mu(\delta\phi^A) = \partial_\mu(\epsilon^a(x) I^A_{(a)B}\phi^B) = \partial_\mu\epsilon^a(x) I^A_{(a)B}\phi^B + \epsilon^a(x) I^A_{(a)B}\partial_\mu\phi^B$$

$$\delta\mathcal{L}_M = \frac{\partial\mathcal{L}_M}{\partial\phi^A}\epsilon^a(x) I^A_{(a)B}\phi^B + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\partial_\mu\epsilon^a(x) I^A_{(a)B}\phi^B + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)}\epsilon^a(x) I^A_{(a)B}\partial_\mu\phi^B$$

$$\delta\mathcal{L}_M = \left[\frac{\partial\mathcal{L}_M}{\partial\phi^A} I^A_{(a)B}\phi^B + \frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)} I^A_{(a)B}\partial_\mu\phi^B\right]\epsilon^a(x) + \left[\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)} I^A_{(a)B}\phi^B\right]\partial_\mu\epsilon^a(x)$$

$$\delta\mathcal{L}_M = \left[\frac{\partial\mathcal{L}_M}{\partial(\partial_\mu\phi^A)} I^A_{(a)B}\phi^B\right]\partial_\mu\epsilon^a(x) \neq 0$$

$A'^J(x) \quad J = 1,2,\dots,M$ → $\delta A'^J = \epsilon^a(x) U^J_{(a)K} A'^K + \frac{1}{g} C^{J\mu}_a \partial_\mu \epsilon^a(x)$

$$\mathcal{L}'_M = \mathcal{L}'_M\left[\phi^A, \partial_\mu \phi^A, A'^J\right]$$

$$\delta\mathcal{L}'_M = \frac{\partial \mathcal{L}'_M}{\partial \phi^A}\delta\phi^A + \frac{\partial \mathcal{L}'_M}{\partial\left(\partial_\mu \phi^A\right)}\delta\left(\partial_\mu \phi^A\right) + \frac{\partial \mathcal{L}'_M}{\partial A'^J}\delta A'^J$$

$$\delta\mathcal{L}'_M = \left[\frac{\partial \mathcal{L}'_M}{\partial \phi^A} I^A_{(a)B}\phi^B + \frac{\partial \mathcal{L}'_M}{\partial\left(\partial_\mu \phi^A\right)} I^A_{(a)B}\partial_\mu\phi^B + \frac{\partial \mathcal{L}'_M}{\partial A'^J} U^J_{(a)K} A'^K\right]\epsilon^a(x) + \left[\frac{\partial \mathcal{L}'_M}{\partial\left(\partial_\mu \phi^A\right)} I^A_{(a)B}\phi^B + \frac{1}{g}\frac{\partial \mathcal{L}'_M}{\partial A'^J} C^{J\mu}_a\right]\partial_\mu \epsilon^a(x)$$

$$\frac{\partial \mathcal{L}'_M}{\partial \phi^A} I^A_{(a)B}\phi^B + \frac{\partial \mathcal{L}'_M}{\partial(\partial_\mu \phi^A)} I^A_{(a)B}\partial_\mu \phi^B + \frac{\partial \mathcal{L}'_M}{\partial A'^J} U^J_{(a)K} A'^K = 0$$

$$\frac{\partial \mathcal{L}'_M}{\partial(\partial_\mu \phi^A)} I^A_{(a)B}\phi^B + \frac{1}{g}\frac{\partial \mathcal{L}'_M}{\partial A'^J} C^{J\mu}_a = 0$$

$$C^{1\mu}_a \frac{\partial \mathcal{L}'_M}{\partial A'^1} + C^{2\mu}_a \frac{\partial \mathcal{L}'_M}{\partial A'^2} + \cdots + C^{M\mu}_a \frac{\partial \mathcal{L}'_M}{\partial A'^M} = -g \frac{\partial \mathcal{L}'_M}{\partial(\partial_\mu \phi^A)} I^A_{(a)B}\phi^B$$

Em coordenadas espaço-tempo:

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{10} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{20} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N0} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_0 \Phi^A)} I_{(a)B}^A \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{11} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{21} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N1} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_1 \Phi^A)} I_{(a)B}^A \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{12} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{22} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N2} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_2 \Phi^A)} I_{(a)B}^A \Phi^B$$

$$\frac{\partial \mathcal{L}'}{\partial A'^1} C_a^{13} + \frac{\partial \mathcal{L}'}{\partial A'^2} C_a^{23} + \cdots + \frac{\partial \mathcal{L}'}{\partial A'^N} C_a^{N3} = -g \frac{\partial \mathcal{L}'}{\partial(\partial_3 \Phi^A)} I_{(a)B}^A \Phi^B$$

$$\begin{bmatrix} C_1^{10} & C_1^{20} & & C_1^{N0} \\ C_1^{11} & C_1^{21} & \cdots & C_1^{N1} \\ C_1^{12} & C_1^{12} & & C_1^{N2} \\ \vdots & & \ddots & \vdots \\ C_n^{13} & C_n^{23} & \cdots & C_n^{N3} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathcal{L}'}{\partial A'^1} \\ \frac{\partial \mathcal{L}'}{\partial A'^2} \\ \vdots \\ \frac{\partial \mathcal{L}'}{\partial A'^N} \end{bmatrix} = -g \begin{bmatrix} \frac{\partial \mathcal{L}'}{\partial(\partial_0 \Phi^A)} I_{(1)B}^A \Phi^B \\ \frac{\partial \mathcal{L}'}{\partial(\partial_1 \Phi^A)} I_{(1)B}^A \Phi^B \\ \vdots \\ \frac{\partial \mathcal{L}'}{\partial(\partial_3 \Phi^A)} I_{(n)B}^A \Phi^B \end{bmatrix}$$

$$C_{4nxN}\,[\vdots]_N = -g[\vdots]_{4n}$$

$$4n = N$$

$$C_a^{J\mu}(C^-)^a_{\mu K} = \delta^J_K$$

$$(C^-)^a_{\nu J} C_b^{J\mu} = \delta^a_b \delta^\mu_\nu$$

$$\delta\left((C^-)^b_{\nu J} A'^J\right) = \epsilon^a(x)(C^-)^b_{\nu J} U^J_{(a)K} A'^K + \frac{1}{g}(C^-)^b_{\nu J} C_a^{J\mu} \partial_\mu \epsilon^a(x) = \delta A^b_\nu$$

$$A'^K = C_c^{K\mu} A^c_\mu$$

$$\delta A^b_\nu = \epsilon^a(x)(C^-)^b_{\nu J} U^J_{(a)K} C_c^{K\mu} A^c_\mu + \frac{1}{g}(C^-)^b_{\nu J} C_a^{J\mu} \partial_\mu \epsilon^a(x)$$

$$\delta A^a_\mu = \epsilon^b(x) S^{a\nu}_{bc\mu} A^c_\nu + \frac{1}{g} \partial_\mu \epsilon^a(x)$$

$$\mathcal{L}\left[\Phi^{A}, \partial_\mu \Phi^{A}\right] \longrightarrow \mathcal{L}'\left[\Phi^{A}, \partial_\mu \Phi^{A}, A'^{J}\right] \longrightarrow \mathcal{L}''\left[\Phi^{A}, \partial_\mu \Phi^{A}, A^a_\mu\right]$$

$$\epsilon^a(x)\left[\frac{\partial \mathcal{L}''}{\partial \Phi^{A}} I^{A}_{(a)B}\Phi^B + \frac{\partial \mathcal{L}''}{\partial A^b_\mu} S^{b\nu}_{ac\mu} A^c_\nu + \frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} I^{A}_{(a)B}\partial_\mu \Phi^B\right] +$$

$$\partial_\mu \epsilon^a(x)\left[\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} I^{A}_{(a)B}\Phi^B + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A^a_\mu}\right] = 0$$

$$\frac{\partial \mathcal{L}''}{\partial \Phi^{A}} I^{A}_{(a)B}\Phi^B + \frac{\partial \mathcal{L}''}{\partial A^b_\mu} S^{b\nu}_{ac\mu} A^c_\nu + \frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} I^{A}_{(a)B}\partial_\mu \Phi^B = 0$$

$$\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} I^{A}_{(a)B}\Phi^B + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A^a_\mu} = 0$$

$$\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\frac{\partial \mathcal{L}''}{\partial A^{a}_{\mu}} = 0$$

$$\nabla_\mu \Phi^{A} = \partial_\mu \Phi^{A} - g A^{a}_{\mu} I^{A}_{(a)B}\Phi^{B}$$

$$\frac{\partial \mathcal{L}''}{\partial(\partial_\mu \Phi^{A})} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{B})}\frac{\partial(\nabla_\nu \Phi^{B})}{\partial(\partial_\mu \Phi^{A})} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{B})}\delta^{\mu}_{\nu}\delta^{B}_{A} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\mu \Phi^{A})}$$

$$\frac{\partial \mathcal{L}''}{\partial A^{a}_{\mu}} = \frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{A})}\frac{\partial(\nabla_\nu \Phi^{A})}{\partial A^{a}_{\mu}} = -g\frac{\partial \mathcal{L}''}{\partial(\nabla_\nu \Phi^{A})}\delta^{b}_{a}\delta^{\mu}_{\nu} I^{A}_{(b)B}\Phi^{B} = -g\frac{\partial \mathcal{L}''}{\partial(\nabla_\mu \Phi^{A})} I^{A}_{(a)B}\Phi^{B}$$

$$\frac{\partial \mathcal{L}''}{\partial(\nabla_\mu \Phi^{A})}\left[I^{A}_{(a)B}\Phi^{B} + \frac{1}{g}\left(-g I^{A}_{(a)B}\Phi^{B}\right)\right] = 0$$

$$\mathcal{L}\left[\Phi^{\mathrm{A}}, \partial_\mu \Phi^{\mathrm{A}}\right] \longrightarrow \mathcal{L}''\left[\Phi^{\mathrm{A}}, \partial_\mu \Phi^{\mathrm{A}}, A_\mu^a\right] \longrightarrow \mathcal{L}_{int}\left[\Phi^{\mathrm{A}}, \nabla_\mu \Phi^{\mathrm{A}}\right]$$

$$\delta\left(\nabla_\mu \Phi^{\mathrm{A}}\right) = \partial_\mu \left(\delta\Phi^{\mathrm{A}}\right) - g\left(\delta A_\mu^a\right) I_{(a)B}^{A} \Phi^B - g A_\mu^a I_{(a)B}^{A} (\delta\Phi^B)$$

$$\delta\left(\nabla_\mu \Phi^{\mathrm{A}}\right) = \epsilon^a(x)\, I_{(a)B}^{A} \partial_\mu \Phi^B - g\epsilon^b(x) S_{bc\mu}^{a\nu} A_\nu^c I_{(a)B}^{A} \Phi^B - g\epsilon^a(x) A_\mu^b I_{(b)B}^{A} I_{(a)C}^{B} \Phi^C$$

$$I_{(a)B}^{A} I_{(b)C}^{B} - I_{(b)B}^{A} I_{(a)C}^{B} = f_{ab}^{c} I_{(c)C}^{A}$$

$$\delta\left(\nabla_\mu \Phi^{\mathrm{A}}\right) = \epsilon^a(x)\, I_{(a)B}^{A} \left[\partial_\mu \Phi^B - g A_\mu^b I_{(b)C}^{B} \Phi^C\right] - g\epsilon^b(x)\left[S_{bc\mu}^{a\nu} - \delta_\mu^\nu f_{bc}^{a}\right] A_\nu^c I_{(a)B}^{A} \Phi^B$$

$$\delta\left(\nabla_\mu \Phi^{\mathrm{A}}\right) = \epsilon^a(x)\, I_{(a)B}^{A} \nabla_\mu \Phi^{\mathrm{B}} - g\epsilon^b(x)\left[S_{bc\mu}^{a\nu} - \delta_\mu^\nu f_{bc}^{a}\right] A_\nu^c I_{(a)B}^{A} \Phi^B$$

$$\frac{\partial \mathcal{L}_{int}}{\partial \Phi^A} \epsilon^a(x)\, I^A_{(a)B} \Phi^B + \frac{\partial \mathcal{L}_{int}}{\partial \left(\nabla_\mu \Phi^A\right)} \epsilon^a(x)\, I^A_{(a)B} \nabla_\mu \Phi^B -$$

$$S^{a\nu}_{bc\mu} = \delta^\nu_\mu f^a_{bc}$$

$$\frac{\partial \mathcal{L}_{int}}{\partial \left(\nabla_\mu \Phi^A\right)} g \epsilon^b(x) \left[S^{a\nu}_{bc\mu} - \delta^\nu_\mu f^a_{bc}\right] A^c_\nu I^A_{(a)B} \Phi^B = 0$$

$$\frac{\partial \mathcal{L}_{int}}{\partial \Phi^A} I^A_{(a)B} \Phi^B + \frac{\partial \mathcal{L}_{int}}{\partial \left(\nabla_\mu \Phi^A\right)} I^A_{(a)B} \nabla_\mu \Phi^B = 0$$

$$\delta A^a_\mu = \epsilon^b(x) f^a_{bc} A^c_\mu + \frac{1}{g} \partial_\mu \epsilon^a(x)$$

Campo de Gauge

$$\mathcal{L}_A \longrightarrow \mathcal{L}_A\left[A_\mu^a, \partial_\nu A_\mu^a\right]$$

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,\delta A_\mu^a + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)}\,\partial_\nu\left(\delta A_\mu^a\right) = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,\epsilon^b(x) f_{bc}^a A_\mu^c + \frac{1}{g}\partial_\mu\,\epsilon^a(x)\frac{\partial \mathcal{L}_A}{\partial A_\mu^a} + \epsilon^b(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a \partial_\nu A_\mu^c +$$

$$\partial_\nu\,\epsilon^b(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a A_\mu^c + \frac{1}{g}\partial_\nu\,\partial_\mu\,\epsilon^a(x)\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} = 0$$

$$\epsilon^b(x)\left[\frac{\partial \mathcal{L}_A}{\partial A_\mu^a}\,f_{bc}^a A_\mu^c + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a \partial_\nu A_\mu^c\right] + \partial_\nu\,\epsilon^b(x)\left[\frac{1}{g}\frac{\partial \mathcal{L}_A}{\partial A_\nu^b} + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)} f_{bc}^a A_\mu^c\right] +$$

$$\partial_\nu\,\partial_\mu\,\epsilon^a(x)\frac{1}{2g}\left[\frac{\partial \mathcal{L}_A}{\partial\left(\partial_\mu A_\nu^a\right)} + \frac{\partial \mathcal{L}_A}{\partial\left(\partial_\nu A_\mu^a\right)}\right] = 0$$

Equações Hierárquicas:

$$\frac{\partial \mathcal{L}_A}{\partial A_\mu^a} f_{bc}^a A_\mu^c + \frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} f_{bc}^a \partial_\nu A_\mu^c = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} f_{bc}^a A_\mu^c + \frac{1}{g}\frac{\partial \mathcal{L}_A}{\partial A_\nu^b} = 0$$

$$\frac{\partial \mathcal{L}_A}{\partial(\partial_\mu A_\nu^a)} + \frac{\partial \mathcal{L}_A}{\partial(\partial_\nu A_\mu^a)} = 0$$

$$\mathcal{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{bc}^a A_\mu^b A_\nu^c$$

$$\delta \mathcal{F}_{\mu\nu}^a = \epsilon^b(x) f_{bc}^a \mathcal{F}_{\mu\nu}^c$$

$$\mathcal{L}_A \longrightarrow \mathcal{L}_A\left[A_\mu^a, \partial_\nu A_\mu^a\right] \longrightarrow \mathcal{L}'_A\left[\mathcal{F}_{\mu\nu}^a\right]$$

$$\frac{\partial \mathcal{L}'_A}{\partial \mathcal{F}_{\mu\nu}^a} f_{bc}^a \mathcal{F}_{\mu\nu}^c = 0$$

$$\frac{\partial \mathcal{L}'_A}{\partial A_\mu^a} = 0$$

Grupo de Transformações U(1)

$$\phi \longrightarrow e^{i\epsilon}\phi$$

$$\phi' = e^{i\epsilon}\phi \cong \phi + i\epsilon\phi$$

$$\phi^* \longrightarrow e^{-i\epsilon}\phi^*$$

$$\phi'^* = e^{-i\epsilon}\phi^* \cong \phi^* - i\epsilon\phi^*$$

$$\begin{pmatrix}\phi' \\ \phi'^*\end{pmatrix} = \begin{pmatrix}\phi \\ \phi^*\end{pmatrix} + \epsilon\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix}\begin{pmatrix}\phi \\ \phi^*\end{pmatrix}$$

$$\left[I_{(a)}, I_{(b)}\right] = 0 \longrightarrow f_{bc}^a = 0$$

Transformação Local

$$\delta A_\mu^a = \epsilon^b(x) f_{bc}^a A_\mu^c + \frac{1}{g}\partial_\mu \epsilon^a(x) \longrightarrow \delta A_\mu = \frac{1}{g}\partial_\mu \epsilon(x)$$

$$\delta \mathcal{F}_{\mu\nu}^a = \epsilon^b(x) f_{bc}^a \mathcal{F}_{\mu\nu}^c \longrightarrow \delta \mathcal{F}_{\mu\nu} = 0$$

$$\mathcal{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{bc}^a A_\mu^b A_\nu^c \longrightarrow \mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\nabla_\mu \Phi^A = \partial_\mu \Phi^A - g A_\mu^a I_{(a)B}^A \Phi^B$$

$$\nabla_\mu \phi = \partial_\mu \phi - i g A_\mu \phi$$

$$\nabla_\mu \phi^* = \partial_\mu \phi^* + i g A_\mu \phi^*$$

Gracias