Matéria Nuclear en Condiciones Extremas

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VII Meeting of Former UNSA Students of Physics 2022 PERUVIAN PHYSICISTS WORLDWIDE

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"It does not seem satisfactory to me to appeal to experience; after all, our purpose in theoretical physics is not just to describe the world as we find it, but to explain – in terms of a few fundamental principles – **why** the world is the way it is."

Steven Weinberg (1933–2021), *The Quantum Theory of Fields*, Volume I: Foundations (1995).

Visión General

Motivaciónes

- 2 Cromodinámica Cuántica (QCD)
- 3 QCD en equilibrio
- 4 QCD fuera del equilibrio
- **(QGP)** ¿Realmente fue descubierto el plasma de quarks/gluones (QGP)?

6 Comentarios Finales

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Preguntas "sencillas" [Wilczek, Phys. Today, 2000]

- P1: ¿Qué pasaria si calentamos algo más y más?
- P2: ¿Qué pasaria si comprimimos algo más y más?
 - R12: Materia Nuclear (Quarks) *Caliente* y *Densa*, respectivamente.



1. Motivación Teórica



José C. Jiménez (IF - USP) Matéria Nue

Motivación Astrofísica



Matéria Nuclear formada en la Evolución Estelar

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Pulsating Sources of Radiation \rightarrow Pulsars



Nebulosa del Cangrejo en diferentes frecuencias electromagnéticas

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Motivaciones experimentales



Relativistic Heavy Ion Collider (RHIC) at BNL [www.bnl.gov/RHIC]

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Evolución de una colisión de iones pesados



Colisión de iones pesados, su interacción y expansión [www.physics.mcgill.ca/music]

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Experimento ALICE en el LHC (CERN)



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Primeras observaciones del "Plasma" de Quarks y Gluones



Physics -

NEWS RHIC groups serve up 'perfect' liquid 5 May 2005

The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory have announced results indicating that they have observed a state of hot, dense matter that is more remarkable than had been predicted. In papers summarizing the first three years of RHIC findings, to be published simultaneously by the journal Nuclear Physics A, the four collaborations (BRAHMS, PHENIX, PHOBOS and STAR) say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy-ion collisions appears to be more like a liquid.

'Perfect' Liquid Hot Enough to be Quark Soup

Protons, neutrons melt to produce 'quark-gluon plasma' at RHIC

February 15, 2010

UPTON, NY - Recent analyses from the Relativistic Heavy Ion Collider (RHIC), a 2.4-mile-circumference "atom smasher" at the U.S. Department of Energy's (DOE) Brookhaven National Laboratory, establish that collisions of gold ions traveling at nearly the speed of light have created matter at a temperature of about 4 trillion degrees Celsius - the hottest temperature ever reached in a laboratory, about 250,000 times hotter than the center of the Sun. This temperature, based upon measurements by the PHENIX collaboration at RHIC, is higher than the temperature needed to melt protons and neutrons into a plasma of quarks and gluons. Details of the findings will be published in Physical Review Letters.

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2. Cromodinámica Cuántica (QCD)



La teoría fundamental de las interacciones fuertes es rica en lo que describe pero "simple" en su formulación.

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***Propiedad Notable:** Libertad Asintótica PREMIO NOBEL DE FÍSICA 2004 - Wilczek/Gross and Politzer



*Propiedad Notable (?): Confinamiento de Color



Masas de hadrones leves y pesados [Durr et al., Science (2008)]

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Uno de los 6 Problemas del Milenio No Resueltos



ABOUT PROGRAMS

MILLENNIUM PROBI

PROBLEMS PEOPLI

PUBLICATIONS EVENTS EUCLI

Yang-Mills and Mass Gap



The laws of quantum physics stand to the world of elementary particles in the way that Newton's laws of classical mechanics stand to the macroscopic world. Almost half a century ago, Yang and Mills introduced a remarkable new framework to describe elementary particles using structures that also occur in geometry. Quantum Yang-Mills theory is now the foundation of most of elementary particle theory, and its predictions have been tested at many experimental laboratories, but its mathematical foundation is still unclear. The successful use of Yang-Mills theory to describe the strong interactions of elementary particles depends on a subtle quantum

mechanical property called the "mass gap": the quantum particles have positive masses, even though the classical waves travel at the speed of light. This property has been discovered by physicists from experiment and confirmed by computer simulations, but it still has not been understood from a theoretical point of view. Progress in establishing the existence of the Yang-Mills theory and a mass gap will require the introduction of fundamental new ideas both in physics and in mathematics.

Rules: Rules for the Millennium Prizes most Status of the Problem Description In Status of the Problem by Michael Douglas Related Links: Lecture by Lorenzo Sadun

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This problem is:

Unsolved

[Clay Mathematics Institute, 2020]

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3. QCD en Equilibrio Termodinámico

• La presión del sistema (en equilibrio térmico) compuesto por quarks y gluones en interacción, *en principio* debería ser obtenida a partir de

$$\mathsf{P}_{\mathrm{QCD}}(\mathsf{T}, \{\mu_i\}, \{\mathsf{m}_i\}) = -\frac{\mathsf{T}}{\mathsf{V}} \log \int \mathcal{D} \bar{\psi} \mathcal{D} \psi \mathcal{D} \mathsf{A}_{\mu} e^{-\int d^3 x \int_0^{1/\mathsf{T}} d\tau \mathcal{L}_{\mathrm{QCD}}^{\mathsf{E}}},$$

donde

$$\mathcal{L}_{\text{QCD}}^{E} = \frac{1}{4} F_{\mu\nu}^{a} F_{\mu\nu}^{a} + \bar{\psi}_{i} (\gamma_{\mu} D_{\mu} + m_{i} - \mu_{i} \gamma_{0}) \psi_{i},$$

y

$$\begin{split} F^{a}_{\mu\nu} &= \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + g_{B}f^{abc}A^{b}_{\mu}A^{c}_{\nu}, \\ D_{\mu} &= \partial_{\mu} - ig_{B}A_{\mu}, \quad A_{\mu} = A^{a}_{\mu}T^{a}. \end{split}$$

• Usando $P_{\rm QCD}$, y atravez de una transformación de Legendre, uno puede obtener la densidad de energía asociada dada por

$$\epsilon_{\rm QCD} = -P_{\rm QCD} + \sum_{i} \mu_i \frac{\partial P_{\rm QCD}}{\partial \mu_i} + T \frac{\partial P_{\rm QCD}}{\partial T}$$

Ej. 1: Lattice QCD para T finito y $\mu_B = 0$



[Borsanyi et al., arXiv:1309.5258]

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Ej. 2: QCD perturbativa densa y fria (pQCD)



$$P(\mu_B)/P_{\text{free}} \sim 1 + \underbrace{c_1 g^2}_{NLO} + \underbrace{c_2 g^4 + c_2' g^4 \log g}_{NNLO} + \underbrace{c_3' g^6 \log^2 g + c_3'' g^6 \log g + \dots}_{N^3 LO}$$

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Ej. 2: Calculos más recientes de pQCD



Matéria QCD extrema.

Panel izquierdo: Caso a temperatura finita en diferentes formalismos, Panel derecho: Matéria pQCD neutra y equilibrio beta, [Fraga *et al.*, 2014].

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Laboratório para el equilibrio denso y frio: Estrellas de Neutrones



Interior (hipotético) de las Estrellas de Neutrones [Weber, 2000]

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Masas y Radios de Estrellas de Neutrones

Asumiendo que el interior de la estrella de neutrones posee simetría esférica y es estacionaria,

$$ds^2 = -e^{\lambda(r)}dt^2 + e^{\nu(r)}dr^2 + r^2(d\theta^2 + sen^2\theta d\phi^2),$$

al usar las ecuaciones de Einstein,

$$G^{\mu\nu} = -8\pi G T^{\mu\nu}_{FP},$$

y asumiendo que el interior estelar es un fluido perfecto,

$$T_{FP}^{\mu\nu} = Pg^{\mu\nu} + (\epsilon + P)u^{\mu}u^{\nu},$$

uno puede obtener la ecuaciones de equilibrio hidrostático conocidas como de Tolman-Oppenheimer-Volkov (TOV)

$$\frac{d\mathcal{M}}{dr} = 4\pi r^{2}\epsilon,$$

$$\frac{dP}{dr} = -\frac{G\mathcal{M}\epsilon}{r^{2}} \left(1 + \frac{P}{\epsilon}\right) \left(1 + \frac{4\pi r^{3}P}{\mathcal{M}}\right) \left(1 - \frac{2G\mathcal{M}}{r}\right).$$

Microfísica de las Estrellas de Neutrones

• La microfísica existente en el interior de las estrellas de neutrones es caracterizada por su ecuación de estado (EoS)

$$P=P(\epsilon),$$

donde *P* y ϵ dependen de μ_i , *T*, *B*, μ_I , μ_5 ,

- La EoS lleva consigo toda la información termodinámica en equilibrio para determinar la estructura y además dinámica, etc.
- **Ejemplo:** Para resolver las ecs. TOV, uno requiere como ingrediente fundamental la EoS para materia nuclear/quarks fria y densa.

Diagrama Masa-Radio para Estrellas de Neutrones



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4. QCD Fuera del Equilibrio Termodinámico



El antes, durante, y después de la colisión [C. Shen, //u.osu.edu/vishnu]

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QCD Fuera del Equilibrio Termodinámico

• La termodínamica del no equilíbrio, *en principio*, se debe obtener a partir de la ecuación de Boltzmann de transporte

$$p^{\mu}\partial_{\mu}f = -\mathcal{C}[f], \quad f(\vec{p},t,\vec{x}) \propto rac{dN}{d^3pd^3x},$$

donde C es el funcional de interacción/colisión.

Ej.: Equilíbrio global implicaria f(p, t, x) = f_{eq}(p), i.e. la distribución de Maxwell-Boltzmann que + Leyes de Conservacíon produce las Ec. de Euler de fluidos no viscosos [Huang, Statistical Mechanics 1987].
No equilíbrio significaria asumir

$$f(p^{\mu}, x^{\mu}) = f_{\mathrm{eq}}\left(rac{p^{\mu}u_{\mu}}{T}
ight) \left[1 + \delta f(p^{\mu}, x^{\mu})
ight]$$

lo que produciria (junto a Leyes de Conservación) la ecs. relativistas viscosas de Navier-Stokes [Romatschke, arXiv:0902.3663]

$$D\epsilon + (\epsilon + P)\partial_{\mu}u^{\mu} - \Pi^{\mu\nu}\nabla(\mu u_{\nu}) = 0,$$

(\epsilon + P)Du^{\alpha} - \nabla^{\alpha}P + \Delta^{\alpha}_{\nu}\partial_{\mu} = 0.

4. QCD Fuera del Equilibrio Termodinámico

- Para el caso de QCD fuera del equilibrio, un análogo de la ec. de Boltzmann es imposible de obtener de forma auto-consistente debido al no conocimiento preciso de C[f_{QCD}] en el sector no lineal donde ocurre el confinamiento de color [Romatschke, arXiv:0902.3663].
- Sin embargo, éste tipo de procesos dinámicos es de interes para, por ejemplo, la transición QCD cosmológica y, especialmente, para la colisión de iones pesados en experimentos de altas energias.
- **Modelos:** Hidrodinámica Relativista Ideal/Viscosa <u>Causal</u> + EoS en equilibrio a partir de Lattice QCD + Leyes de Conservación, i.e.
 - Conservación del Tensor Energía-Momentum

$$\partial_{\mu}T^{\mu\nu}[P,\epsilon,n_B](t,\vec{x})=0,$$

- Conservacíon Número Bariónico,

$$\partial_{\mu}(n_B u^{\mu})=0.$$

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5. Q: ¿Realmente fue descubierto el QGP?

Heavy Ion Collisions: The Big Picture, and the Big Questions

Wit Busza,¹ Krishna Rajagopal,^{1,2} and Wilke van der Schee^{2,3}

¹Laboratory for Nuclear Science and Department of Physics, MIT, 77 Massachusetts Avenue, Cambridge, MA 02139, USA ²Center for Theoretical Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA ³Institute for Theoretical Physics and Center for Extreme Matter and Emergent Phenomena, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands

[Busza et al., arXiv:1802.04801]

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Una gran pregunta (?)

5. Can we obtain an experimental determination, even indirectly, of the temperature of the matter produced in a heavy ion collision at a time at which we can also determine its energy density? If we could, we could obtain an experimental determination of the number of thermodynamic degrees of freedom, the quantity whose increase reflects the liberation of color above the crossover in the QCD phase diagram.

[Busza et al., arXiv:1802.04801]

Formación de materia de quarks sin lugar a dudas (?)



Thermodynamics of hot strong-interaction matter from ultrarelativistic nuclear collisions

Fernando G. Gardim^{® 1,2}, Giuliano Giacalone², Matthew Luzum^{® 3} and Jean-Yves Ollitrault^{® 2} ⊠

Collisions between heavy atomic nuclei at ultrarelativistic energies are carried out at particle colliders to produce the quark-gluon plasma, a state of matter where quarks and gluons are not confined into hadrons, and colour degrees of freedom are liberated. This state is thought to be produced as a transient phenomenon before it fragments into thousands of particles that reach the particle detectors. Despite two decades of investigations, one of the big open challenges is The situation in a nucleus-nucleus collision is slightly different. In equark-gluon plasma is not produced at rest, but with a builtin longitudinal expansion imprinted by the collision dynamics, and detectors cover a limited angular range, so that only a slice of the fluid is seen. The energy of this slice is not conserved, but decreases due to the negative work of pressure forces during the longitudinal expansion. As we shall see, however, the above argument still holds, provided that one replaces the initial energy with the final energy.

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[Gardim et al., Nature Phys. 16 (2020), 615]

Colisión de $Pb^{208} - Pb^{208}$ en el LHC (?)



[Gardim et al., Nature Phys. 16 (2020), 615]

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Resultados de las simulaciones hidrodinámicas



[Gardim et al., Nature Phys. 16 (2020), 615]

Resultados de los ajustes entre $< p_t > y T_{eff}$

- A partir de la comparacion de las simulaciones hidrodinámicas con los datos obtenidos por los detectores, se obtuvo $< p_t >= 3.07 T_{\rm eff}$ como un ajuste.
- Además, experimentalmente se sabia que < p_t >= 681 MeV lo que produce

$$T_{\rm eff} = 222 \pm 9 {
m MeV} = (2.58 \pm 0.10) \times 10^{12} {
m K}.$$

• Y ya que la densidad de entropia "s" está asociada a la multiplicidad por " $dN_{\rm ch}/dy$ ":

$$s(T_{\mathrm{eff}}) = rac{1}{V_{\mathrm{eff}}} rac{S}{N_{\mathrm{ch}}} rac{dN_{\mathrm{ch}}}{dy} = 20 \pm 5 \mathrm{fm}^{-3}.$$

• También para la velocidad del sonido c_s^2 obtiene

$$c_s^2(T_{
m eff}) = rac{d\log < p_t >}{d\log(dN_{
m ch}/d\eta)} = 0.24 \pm 0.04.$$

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Propiedades termodinámicas de la matéria caliente de quarks (?)



[Gardim et al., Nature Phys. 16 (2020), 615]

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Matéria Nuclear en Condiciones Extremas



[Glozman, arXiv:2209.10235]

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[Glozman, arXiv:2209.10235]



[Glozman, arXiv:2209.10235]

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[Glozman, arXiv:2209.10235]

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6. Comentarios Finales

- La matéria nuclear sometida a condiciones extremas de temperatura y densidad en laboratórios astrofísicos y terrestres es útil para diseñar el diagrama de fases de la QCD.
- La QCD aún no es bien entendida teóricamente en sectores de interés experimental, ya sea en el vacío ó como un medio térmico.
- Sólo modelos efectivos y simulaciones numéricas pesadas nos ofrecen insights y herramientas para explorar el sector altamente no trivial de la QCD donde la transición confinamiento/desconfinamiento ocurre.
- Las estrellas de neutrones son el mejor laboratório astrofísico conocido para estudiar la materia a altísimas densidades y bajas temperaturas, incluso más ahora con el advenimiento de las observaciones de ondas gravitacionales por LIGO y Virgo.

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Advertencia Final



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